

-notes on L' Hôpital's Rule-

Remember this need to know.....I told you to take it on my word that it was true?

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \qquad \frac{0}{0} \text{ is an indeterminant form}$$

L'Hopital's rule states: if the limit is an indeterminant form and g'(x) does not equal 0 then the following holds true:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Main Indeterminant forms: $\frac{0}{0}, \frac{\infty}{\infty}, \frac{-\infty}{\infty}, \frac{\infty}{-\infty}, \frac{-\infty}{-\infty}$

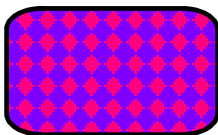
Let's try this: pay attention to the notation:

Other indeterminant forms: $0 \cdot \infty, 1^\infty, 0^0$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \overset{0}{\underset{0}{\frac{0}{0}}} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 0} \frac{(1+x)^{1/2} - 1}{x} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0} \frac{\frac{1}{2}(1+x)^{-1/2}}{1} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0} \frac{1}{2\sqrt{1+x}} = \frac{1}{2}$$

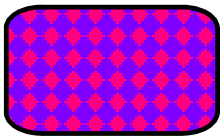
$$\lim_{x \rightarrow 0} \frac{1}{2\sqrt{1+x}} = \frac{1}{2}$$



$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0^+} \frac{\cos x}{2x} \quad \frac{1}{2 \cdot (0^+)} \rightarrow \infty$$

$$\lim_{x \rightarrow 0^-} \frac{\sin x}{x^2} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0^-} \frac{\cos x}{2x} \quad \frac{1}{2 \cdot (0^-)} \rightarrow -\infty$$

$x \rightarrow 0^+ \rightarrow .000001$



$$\lim_{x \rightarrow \infty} \frac{\ln x}{2\sqrt{x}} \xrightarrow{\text{L'H}} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{2}{\sqrt{x}}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{2x} = \lim_{x \rightarrow \infty} \frac{1}{2\sqrt{x}} \rightarrow 0$$

$$\lim_{x \rightarrow \infty} \left(x \sin \frac{1}{x} \right) \quad \infty \cdot 0$$

$$\lim_{x \rightarrow \infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}} \quad \frac{0}{0}$$

$$\xrightarrow{LH} \lim_{x \rightarrow \infty} \frac{-\frac{1}{x^2} \cos \frac{1}{x}}{-\frac{1}{x^2}} = 1$$

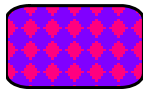
$$\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right) \quad \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{x-1 - \ln x}{(\ln x)(x-1)} \quad \xrightarrow{LH}$$

$$\lim_{x \rightarrow 1} \frac{1 - \frac{1}{x}}{\frac{1}{x}(x-1) + \ln x} \quad \frac{0}{0}$$

$$\xrightarrow{LH} \lim_{x \rightarrow 1} \frac{\frac{1}{x^2}}{\frac{1}{x^2} + \frac{1}{x}} = \boxed{\frac{1}{2}}$$

$$\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2} = -4 \frac{(x+2)(x-2)}{(x+2)}$$



$$\ln y = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x = e$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{1}{x} \right)}{\frac{1}{x}} \quad \frac{0}{0}$$

$$\xrightarrow{LH}$$

$$\lim_{x \rightarrow \infty} \frac{x}{x+1} = 1$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{-\frac{1}{x^2}}{\frac{x+1}{x} - \frac{1}{x^2}}$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{-\frac{1}{x^2} \cdot \frac{x}{x+1}}{\frac{1}{x^2}}$$

$$\lim_{x \rightarrow \infty} \frac{1}{1} = 1$$

$\ln y = 1$
 $e = e$
 $y = e$

Hwk # 50

9. 2/1, 2, 5, 6, 27, 35-42 all, 58

Set 78/1-19 odd