-notes on L' Hôpital's Rule-

Remember this need to know.....I told you to take it on my word that it was true?

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$\frac{0}{0}$ is an indeterminant form

L'Hopital's rule states: if the limit is an indeterminant form and g'(x) does not equal 0 then the following holds true:

$$\boxed{\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}}$$

Main Indeterminant forms: $\frac{0}{0},\frac{\infty}{\infty},\frac{-\infty}{\infty},\frac{\infty}{-\infty},\frac{-\infty}{-\infty}$

Let's try this: pay attention to the notation:

Other indeterminant forms: $0\bullet\infty, 1^{\infty}, 0^{0}$

$$\lim_{x\to 0}\frac{\sin x}{x}=\frac{0}{0}\ \xrightarrow{LH}\ \lim_{x\to 0}\frac{\cos x}{1}=\frac{1}{1}=1$$

$$\lim_{x\to 0}\frac{\overset{(1+x)^{1/2}}{\sqrt{1+x}}-1}{x}\quad \frac{0}{0}\ \xrightarrow{LH}\ \lim_{x\to 0}\frac{\frac{1}{2}(1+x)^{-1/2}}{1}\quad \lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+x}}}{1}=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{1}{2\sqrt{1+x}}=\frac{1}{2}$$

$$\lim_{x\to 0^+}\frac{\sin x}{x^2} \quad \frac{0}{0} \quad \xrightarrow{\text{LH}} \lim_{x\to 0^+} \frac{\cos x}{2x} \qquad \frac{1}{2\cdot(0^+)} \to \infty$$

$x \to 0^+$

$-.000001$

$$\lim_{x\to 0^-}\frac{\sin x}{x^2} \quad \frac{0}{0} \quad \xrightarrow{\text{LH}} \lim_{x\to 0^-} \frac{\cos x}{2x} \qquad \frac{1}{2(0^-)} \to -\infty$$

$$\frac{\infty}{\infty}\ \xrightarrow{\text{LH}} \lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}}{1} \to 0$$

$$\lim_{x\to\infty}\frac{\ln x}{2\sqrt{x}} \quad \frac{\infty}{\infty} \quad \xrightarrow{\text{LH}} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{\cancel{2}}{\cancel{2}\sqrt{x}}} = \lim_{x\to\infty} \frac{\sqrt{x}}{x} = \lim_{x\to\infty} \frac{1}{\sqrt{x}} \to 0$$

$\infty \cdot 0$

$$\lim_{x\to\infty}\left(x\sin\frac{1}{x}\right) \qquad \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{x}} \quad \frac{0}{0} \quad \xrightarrow{LH} \lim_{x\to\infty}\frac{\cancel{-\frac{1}{x^2}}\cos\frac{1}{x}}{\cancel{-\frac{1}{x^2}}} = 1$$

$$\lim_{x\to1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right) \qquad \lim_{x\to1}\left(\frac{x-1-\ln x}{(\ln x)(x-1)}\right) \quad \frac{0}{0} \quad \xrightarrow{LH}$$

$$\lim_{x\to1}\frac{1-\frac{1}{x}}{\boxed{\frac{1}{x}(x-1)}+\ln x} \quad \frac{0}{0} \quad \xrightarrow{LH} \lim_{x\to1}\frac{\frac{1}{x^2}}{\frac{1}{x^2}+\frac{1}{x}} = \boxed{\frac{1}{2}}$$

$$\lim_{x\to-2}\frac{x^2-4}{x+2} = -4 \qquad \frac{(\cancel{x+2})(x-2)}{(\cancel{x+2})}$$

$$\ln y = \ln\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

$$\ln y = \lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \quad \frac{0}{0} \quad \xrightarrow{LH} \qquad \lim_{x\to\infty}\frac{x}{x+1} = 1 \quad \frac{\infty}{\infty}$$

$$\ln y = \lim_{x\to\infty}\frac{\frac{-\frac{1}{x^2}}{\frac{x+1}{x}}}{-\frac{1}{x^2}} \qquad \ln y = \lim_{x\to\infty}\frac{\cancel{-\frac{1}{x^2}}\cdot\frac{x}{x+1}}{\cancel{-\frac{1}{x^2}}} \qquad \lim_{x\to\infty}\frac{1}{1} = 1$$

$$e^{\ln y} = e^{1}$$

$$y = e$$

Hwk # 50

9. 2/1, 2, 5, 6, 27, 35-42 all, 58

Set 78/1-19 odd